Nuclear Physics
Final exam
Date: Friday, July 4, 2008
This exam has a total of 100 points

Problem #1. *(5 points)* Assuming that ultra-relativistic energies correspond to a β of 0.95 and more, calculate at which kinetic energies (in units of MeV) an electron and an alpha particle become ultra-relativistic. Take 4 nucleon masses for the mass of the alpha particle.

Problem #2. *(5 points)* Certain radioactive nuclei emit α particles. If the kinetic energy of these α particles is 4 MeV, what is their velocity if you assume them to be non-relativistic? *(2 ponits)*

How large an error do you make in neglecting special relativity in the calculation of this velocity? *(3 points)*

Problem #3. *(6 points)* The existence of a massive meson was already suggested by Yukawa in 1935 (Nobel prize, 1949) to explain the nuclear forces. One knows that the strong interaction has a short range and the force basically only exists within nuclei and acts at short distances. In the theory of Yukawa, a heavy meson can be exchanged between two nucleons. This meson (virtual) can act as the messenger of the strong force then (this is similar to electromagnetic force for which the photon is the messenger and the range is infinite).

a) From the uncertainty principle $\Delta t \cdot \Delta E = \hbar$, derive a relation between the range of the force, r, and the mass of the meson. *(3 points)*

b) In the nuclei, the inter-distance between nucleons is less than 2 fm. Using this number, estimate the mass of this heavy meson (later called a pion). Compare the number with the actual mass of about 140 MeV/c^2. *(3 points)*

Problem #4. *(4 points)*

Show that for elastic scattering of a high-energy electron with energy E_0 and momentum $kc \approx E_0$ with a heavy nucleus, the magnitude of the momentum transfered to the nucleus is $q = 2k\sin(\theta/2)$ where θ is the scattering angle.

Problem #5. *(10 points)*

Calculate the differential cross section, using the following formula for the scattering amplitude in first Born approximation

$$f(\theta) = -\frac{m}{2\pi\hbar^2}\int V(\vec{r})\exp(i\vec{q}\cdot\vec{r}/\hbar)d^3r$$

and assuming a shielded Coulomb potential given by the following form:

$$V(r) = \frac{Z_1 Z e^2}{r}e^{-r/a}$$

where Z_1 and Z are the charges of the beam and target nuclei, respectively and a parameterizes the shielding. Take the limit as $a \to \infty$ and explain your result.

You need the following integral for your calculation:

$$\int_0^\infty \exp(-kr)\sin(qr)dr = \frac{q}{k^2+q^2}$$

Problem #6. *(5 points)*

Nucleus ^{113}Cd absorbs a thermal neutron and goes to the excited state of ^{114}Cd(1^-). What is the energy and the multipolarity of the γ decay to the ground state (0^+)? Note that M(^{113}Cd)=112.9044 and M(^{114}Cd)=113.903357 a.m.u.

Problem #7. *(16 points)*

a) Write down the Bethe-Weizsäcker empirical mass formula (you don't need to know the constants. Here, they are given for your information: $a_V = 15.85$ MeV, $a_S = 18.34$ MeV, $a_a = 23.21$ MeV, $a_c = 0.71$ MeV and $a_p = 12$ MeV). *(4 points)*

b) Give a physical argument as to why the form is parabolic in Z for large Z? Elaborate by making an estimation leading to this term. *(4 points)*

c) Calculate the relation between the mass number A and the charge number Z for the most stable nuclei assuming large A (line of stability). *(4 points)*

d) For a constant *odd* A, draw a picture of the binding energy as a function of Z and show how the stable isotope is reached and through which decay paths (specify the decay type as well). *(4 points)*

Problem #8. *(6 points)* Determine the magnetic dipole moment μ/μ_N of the deuteron for a pure $^3S_1$ state in terms of magnetic dipole moments of proton and neutron. Note that deuteron is made out of one proton and one neutron which couple to $J = 1$ with spins aligned when particles are both in s orbital to form the pure $^3S_1$. *(4 points)* The calculated value is about 2.5% larger than the observed value. What could we learn from this? *(2 points)*

Problem #9. *(8 points)*

a) The nuclear spin-orbit coupling is proportional to the expectation value of $\vec{l}\cdot\vec{s}$. Express $\vec{l}\cdot\vec{s}$ in terms of j, l and s. Show that the energy separation of a nuclear spin-orbit doublet is proportional to $2l+1$ (Note that $s = 1/2$ for this). *(4 points)*

b) Predict the first 4 levels (starting from the ground state) of ^{208}Pb using the level scheme provided. *(4 points)* 82 protons

Problem #10. *(8 points)*

a) Using the expansion of the operator $\exp(i\vec{k}\cdot\vec{r})$, state what electromagnetic transitions correspond to the first two terms in the expansion. *(4 points)*

b) Explain, with simple estimates, why the first term of the transition is, generally, much more important in atomic physics than in nuclear physics. To have the natural units in atomic physics, use $\hbar c \approx 2000$ evÅ. *(4 points)*

Problem #11. *(4 points)*
Why do nuclei and particles not have permanent electric dipole moment? Elaborate on your answer.

Problem #12. *(7 points)* Show mathematically how in the beta-decay experiments, the measurement of a pseudo-scalar observable (such as helicity) can give a proof of parity non-conservation of the wave-functions considered.

Problem #13. *(16 points, 2 points per item)*
The nucleon-nucleon force is usually described as: (a) strong, (b) short-ranged, (c) charge symmetric, (d) charge independent, (e) containing both 'ordinary' and 'exchange' terms, (f) spin dependent, (g) non-central, and (h) saturating in nuclei. Explain the meaning of these statements and discuss briefly the experimental evidence for them.

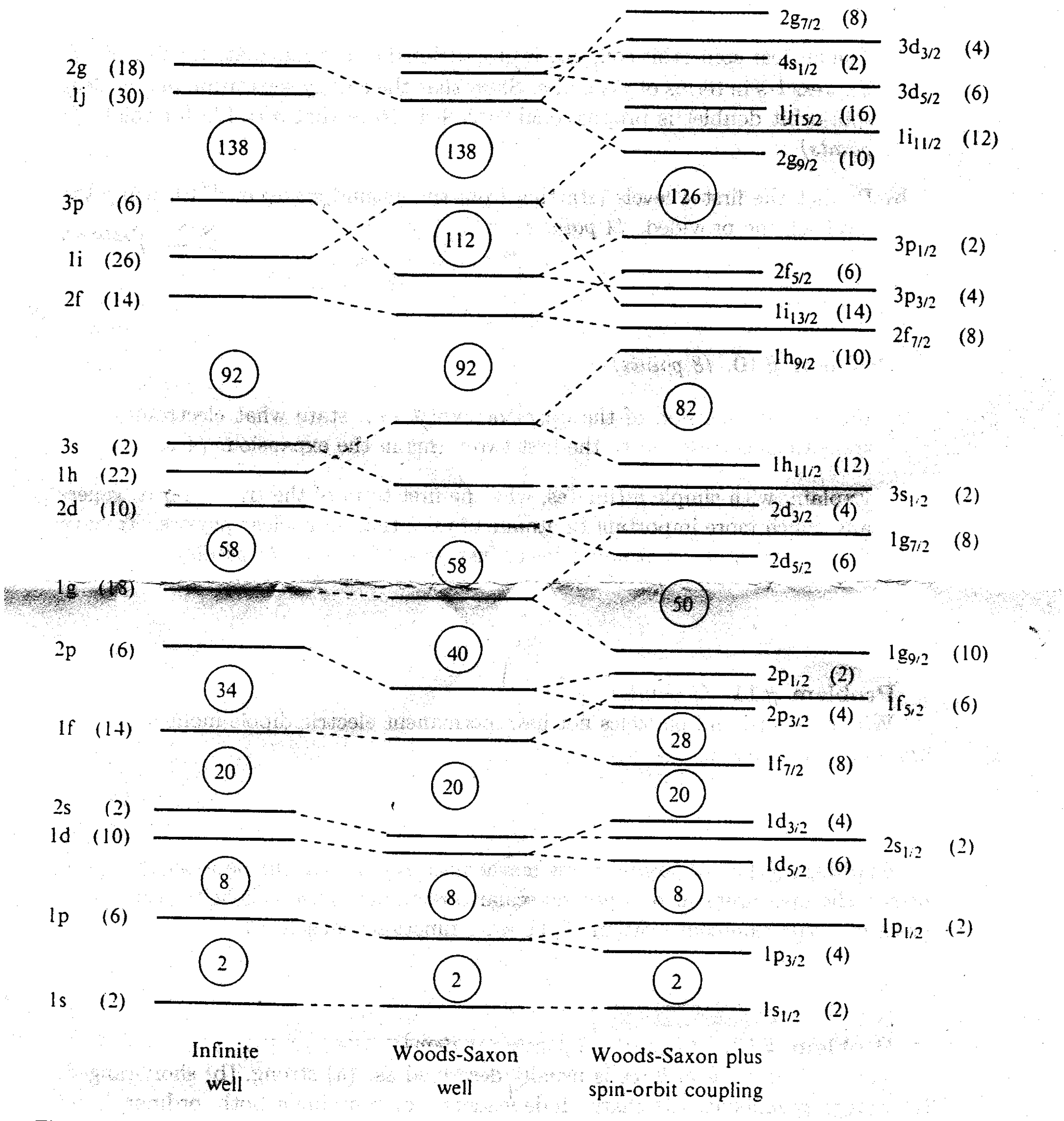

Figure 2.9 Sequences of bound single-particle states calculated for different forms of the nuclear shell-model potential. The number of protons (and neutrons) allowed in each state is indicated in parentheses and the numbers enclosed in circles indicate magic numbers corresponding to closed shells.